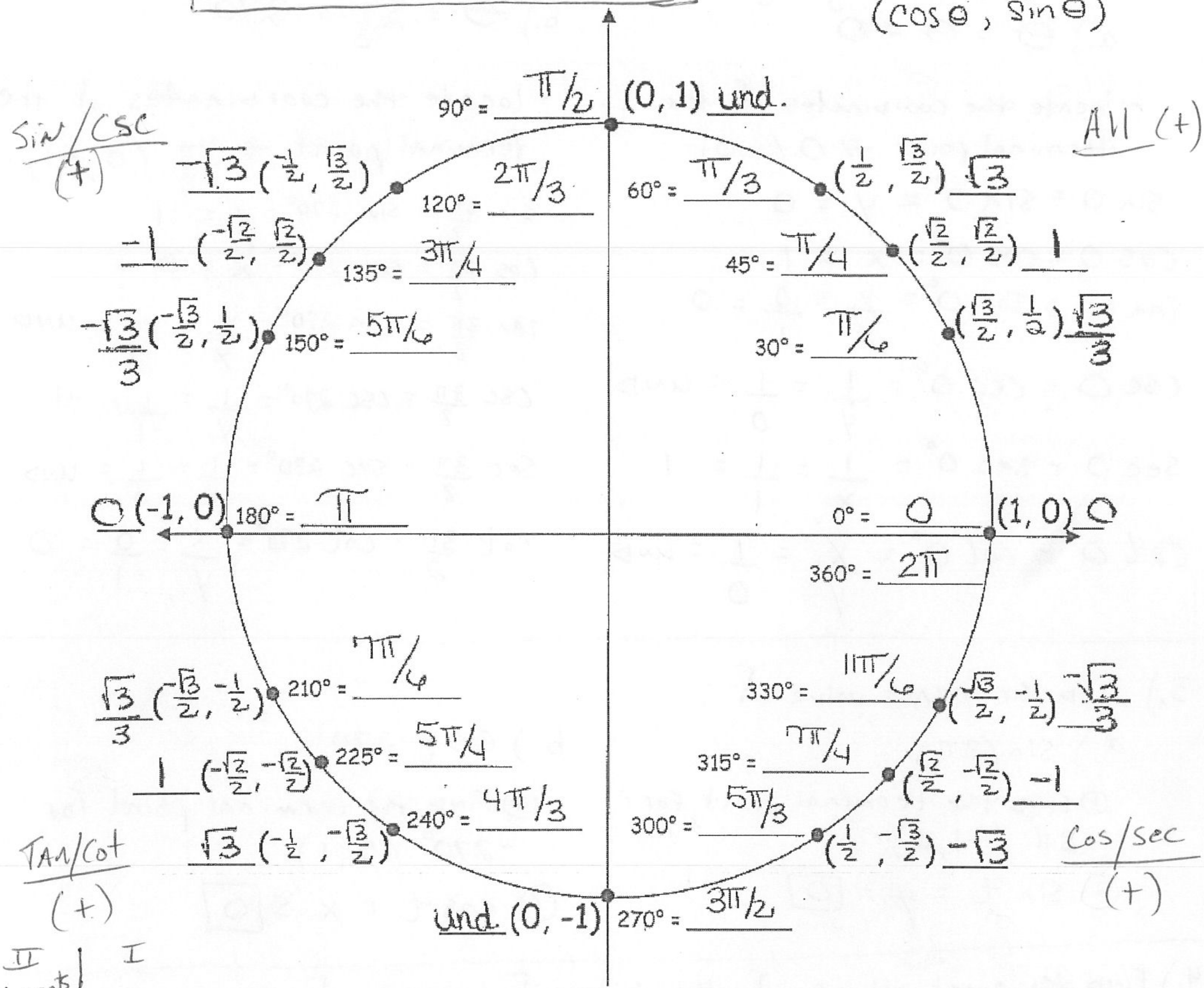
The Unit Circle

(x, y)
$(\cos\theta, \sin\theta)$



II	I
Students	All
Take	Calculus
III	IV

→ How to remember which trig functions are positive in a particular quadrant.

• Let t be a real number where s (arc length) $= |t|$ (Always starting at 0°) and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

$\sin t = y$ $\qquad \csc t = \frac{1}{y},\ y \neq 0$

$\cos t = x$ $\qquad \sec t = \frac{1}{x},\ x \neq 0$

$\tan t = \frac{y}{x},\ x \neq 0$ $\qquad \cot t = \frac{x}{y},\ y \neq 0$

2.) Finding the Exact Values of the Six Trig functions of Quadrantal Angles

a.) $\theta = 0 = 0^\circ$

• locate the coordinates of the terminal point $\rightarrow 0\ (1,0)$

$\sin 0 = \sin 0^\circ = y = 0$

$\cos 0 = \cos 0^\circ = x = 1$

$\tan 0 = \tan 0^\circ = \frac{y}{x} = \frac{0}{1} = 0$

$\csc 0 = \csc 0^\circ = \frac{1}{y} = \frac{1}{0} = \text{UND}$

$\sec 0 = \sec 0^\circ = \frac{1}{x} = \frac{1}{1} = 1$

$\cot 0 = \cot 0^\circ = \frac{x}{y} = \frac{1}{0} = \text{UND}$

b.) $\theta = \frac{3\pi}{2} = 270^\circ$

• locate the coordinates of the terminal point $\rightarrow \frac{3\pi}{2}\ (0,-1)$

$\sin \frac{3\pi}{2} = \sin 270^\circ = y = -1$

$\cos \frac{3\pi}{2} = \cos 270^\circ = x = 0$

$\tan \frac{3\pi}{2} = \tan 270^\circ = \frac{y}{x} = \frac{-1}{0} = \text{UND}$

$\csc \frac{3\pi}{2} = \csc 270^\circ = \frac{1}{y} = \frac{1}{-1} = -1$

$\sec \frac{3\pi}{2} = \sec 270^\circ = \frac{1}{x} = \frac{1}{0} = \text{UND}$

$\cot \frac{3\pi}{2} = \cot 270^\circ = \frac{x}{y} = \frac{0}{-1} = 0$

3.) Find the exact value of:

a.) $\sin(3\pi)$

① Find the terminal point for $3\pi\ (-1,0)$

② $\sin t = y = \boxed{0}$

b.) $\cos(-270)$

① Find the terminal point for $-270^\circ\ (0,1)$

② $\cos t = x = \boxed{0}$

4.) Find the exact values of the 6 trig functions of $\frac{\pi}{4}$

a.) Terminal point of $\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

b.) $\sin \frac{\pi}{4} = y = \frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = x = \frac{\sqrt{2}}{2}$

$\tan \frac{\pi}{4} = \frac{y}{x} = 1$

$\csc \frac{\pi}{4} = \frac{1}{y} = \frac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

$\sec \frac{\pi}{4} = \frac{1}{x} = \frac{1}{\frac{\sqrt{2}}{2}} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

$\cot \frac{\pi}{4} = \frac{x}{y} = \boxed{1}$

5.2 - Trig Functions: The Unit Circle Approach <cont'd>

5.) Find the exact value of each expression.

a.) $\sin 45^\circ \cos 180^\circ = \frac{\sqrt{2}}{2} \cdot (-1) = \boxed{-\frac{\sqrt{2}}{2}}$

b.) $\tan \frac{\pi}{4} - \sin \frac{3\pi}{2} = 1 - (-1) = \boxed{2}$

c.) $\left(\sec \frac{\pi}{4}\right)^2 + \csc \frac{\pi}{2} = (\sqrt{2})^2 + 1 = 2 + 1 = \boxed{3}$

6.) Find the exact values of the following:

a.) $\sin 135^\circ \rightarrow$ terminal pt $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rightarrow \sin 135^\circ = y = \boxed{\frac{\sqrt{2}}{2}}$

b.) $\cos \frac{5\pi}{4} \rightarrow tp \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \rightarrow \cos \frac{5\pi}{4} = x = \boxed{-\frac{\sqrt{2}}{2}}$

c.) $\tan 315^\circ \rightarrow tp \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \rightarrow \tan 315^\circ = \frac{y}{x} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{-1}$

d.) $\cos 210^\circ \rightarrow tp \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \rightarrow \cos 210^\circ = x = \boxed{-\frac{\sqrt{3}}{2}}$

e.) $\sin(-60^\circ) \rightarrow tp \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \rightarrow \sin(-60^\circ) = y = \boxed{-\frac{\sqrt{3}}{2}}$

f.) $\tan \frac{5\pi}{3} \rightarrow tp \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \rightarrow \tan \frac{5\pi}{3} = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{-\sqrt{3}}$

7.) Use a Calculator to approximate the value of:

a.) $\cos 48^\circ \approx \underline{0.67}$

b.) $\csc 21^\circ = \frac{1}{\sin 21^\circ} \approx \underline{2.79}$

c.) $\tan \frac{\pi}{12} \approx \underline{0.27}$

* make sure to set mode to <u>degrees</u> for a+b

* set mode to radians for c.

8.) Use a circle of radius, r to evaluate trig functions

- For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the $x^2 + y^2 = r^2$. Then

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y},\ y \neq 0$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x},\ x \neq 0$$

$$\tan\theta = \frac{y}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

- Find the exact value of the 6 trig functions of an angle θ if $(4, -3)$ is a point on its terminal side.

$$x^2 + y^2 = r^2 \rightarrow (4)^2 + (-3)^2 = r^2 \rightarrow \sqrt{16 + 9} = r \rightarrow \underline{5 = r}$$

$$\sin\theta = \frac{y}{r} = \boxed{\frac{-3}{5}} \qquad \cos\theta = \frac{x}{r} = \boxed{\frac{4}{5}} \qquad \tan\theta = \frac{y}{x} = \boxed{\frac{-3}{4}}$$

$$\csc\theta = \frac{r}{y} = \boxed{\frac{-5}{3}} \qquad \sec\theta = \frac{r}{x} = \boxed{\frac{5}{4}} \qquad \cot\theta = \frac{x}{y} = \boxed{\frac{-4}{3}}$$

9.) Projectile Motion: The path of a projectile fired at an inclination θ to the horizontal with initial speed V_0 is a parabola. The range R of the projectile, that is, the horizontal distance that the projectile travels, is found by using the formula: $R = \frac{(V_0)^2 (\sin 2\theta)}{g}$ where $g \approx 32.2$ Ft/sec or 9.8 Ft/sec is the acceleration due to gravity. The max height H of the projectile is $H = \frac{(V_0)^2 (\sin\theta)^2}{2g}$

- Find the range R + max height H of a projectile fired at a $30°$ to the horizontal w/an initial speed of 150 meters/sec

$$R = \frac{(150)^2 (\sin 2 \cdot 30°)}{9.8} \qquad \boxed{R \approx 1988.32\,m}$$

$$H = \frac{(150)^2 (\sin 30°)^2}{2(9.8)} \qquad \boxed{H \approx 287\,m}$$